Earthquake aftershock networks generated on Euclidean spaces of different fractal geometry

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According to some recent analysis (M. Paczuski and M. Baiesi, Phys. Rev. E **69**, 066106, 2004 [1]) of earthquake data, aftershock epicenters can be considered to represent the nodes of a network where the linking scheme depends on several factors. In the present paper a model network of earthquake aftershock epicenters is proposed based on this scheme and studied on fractals of different dimensions. The various statistical features of this network, like degree, link length, frequency and correlation distributions are evaluated and compared to the observed data. The results are also found to be independent of the fractal geometry.

An earthquake is a complicated spatio-temporal phenomenon [2] which exhibits complex correlation in space, time as well as magnitude. Physically, earthquakes occur when the convective motion in the mantle cause sudden rupture and deformation of certain parts of the earth's crust that result in the emanation of energy in the form of seismic waves. In other words, earthquakes result from the interaction between stress concentration and fluid flow and have recently been the subject of considerable interest in various fields including statistical physics because of its unique statistical features.

Depending on the relative magnitude and position in the space-time sequence we can primarily classify earthquakes into three categories of events [1–3] :
(i) Intermediate or small amplitude precursor events that precede a main event, known as *fore-shocks*.
(ii) Events of relatively large magnitude and impact, known as *main shocks*.
(iii) Nearby smaller correlated events that follow a large seismic event or main shock, known as *aftershocks*.

Earthquakes are best understood by studying large space-time correlations of many events instead of observing isolated individual events and it has been suggested that earthquake is a critical phenomenon [4]. The striking statistical features followed by earthquakes in general are the following:
(a) The distribution of earthquake magnitude (m), is given by the Gutenberg-Richter (GR) law [5]:

$$P(m) \sim 10^{-bm}, (b \sim 1) \qquad (1)$$

where $P(m)$ is the number of earthquakes of magnitude m in a seismic region.
(b) The short time temporal correlation between earthquakes follows the Omori law [6], which states that following a main event, the frequency of a sequence of aftershocks decays with time t as:

$$N(t) \sim t^{-\alpha}, (\alpha \sim 1) \qquad (2)$$

(c) The spatial distribution of earthquake epicenters form a fractal set with fractal dimension d_f [7].
Several analyses of real earthquake data reveal that earthquakes are a result of a dynamical many-body system that reaches a stationary critical state characterised by spatial and temporal correlations that follow power laws (eq.s 1,2) without any intrinsic time or length scales and hence they are related to self organised critical phenomena [8,9].

The Earthquake Network:

In [1] the earthquake aftershock phenomenon has been visualised as a network. Networks are complex web like structures comprising of nodes, connected by links and such structures describe a wide variety of natural systems and have been studied recently with growing interest [10]. In [1] the nodes are the earthquake/aftershock epicenters and they are linked with a weightage given by the correlation between them. For the generation of an earthquake network, a metric is required which would quantify the correlation between two events or whether one event can be considered as an aftershock of another. This metric should incorporate the self similar statistical properties that unify earthquakes in general and is given by [1]:

$$n_{ij} \equiv Ctl^{d_f} \Delta m 10^{-bm_i} \qquad (3)$$

where i and j represent events, i.e, earthquakes and/or aftershocks time ordered with i preceding j.
The various quantities appearing in eq.(3) are as follows:
m_i = Magnitude of the ith event;
$l = l_{ij}$ = Spatial distance between two earthquake epicenters corresponding to the ith and j^{th} events;
$t = t_{ij} = T_j - T_i$ = Time interval between the two events, with $T_j > T_i$;
C = A constant depending on the overall seismicity in the region under consideration and
n_{ij} = the expected number of events with magnitude

within Δm of m_i, occurring within the space-time domain bounded by events i and j.

Of all the earthquakes preceding j, there must be some event $i = i^*$ which is most unlikely to occur and for which n_{ij} is minimum. Nevertheless since i^* actually took place relative to j, inspite of being the most unlikely, so i^* is the event most correlated to j. Hence the degree of correlation between any two earthquakes i and j is inversely proportional to n_{ij} and two events i^* and j are linked if n_{ij} is minimum for $i = i^*$. Thus a network is generated which is directed in time where, the nodes are the earthquake/aftershock epicenters, characterised by internal parameters which are magnitude, location and time of occurrence, and any two events are linked with a weight n_{ij}. In this paper our aim is to find out whether it is possible to construct a model which reproduces results comparable to the observed behaviour of the statistical features of some real earthquake data. We have also studied the dependence of the results on Euclidean spaces of different fractal dimensions.

Observed results for real earthquakes:
In [1], the catalogue maintained by the Southern California Earthquake Data Center (SCEDC) has been analysed. The values of the various constants in eq.(3) that are used are $b \simeq 0.95$, $C = 10^{-9}$.
The value of the fractal dimension used is $d_f \simeq 1.6$ following the result obtained in [11].
The analysis led to the following observations:

(a) The distribution of the time intervals of the occurrence of aftershocks, $\nu(t)$, shows a power law decay ($\nu(t) \sim \frac{K}{c+t}$ for $t < t_{cutoff}$; c and K are constant in time and t_{cutoff} depends on the magnitude m of the earthquake) which is consistent with the Omori law [6].
(b) The resulting network of earthquakes is scale-free, i.e., the out-degree distribution $P(k)$ follows a power law decay, and for the SCEDC data it is found that $P(k) \sim k^{-\gamma}$ with $\gamma = 2.0$.
(c) The link weight distribution $\mathcal{N}(n_{ij})$ exhibits a power law decay with slope $\simeq 1$.
(c) The link length is defined in [1] as the distance between the epicenter of an aftershock and its linked predecessor and its distribution $\mathcal{L}(l)$ shows an approximate power law decay.

Modelling and Results:
In our present work we simulate a network of earthquake aftershocks on Euclidean spaces with different values of the fractal dimension d_f. These are:
(i) Continuous two dimensional Euclidean space ($d_f = 2.0$)
(ii) Percolation cluster on two dimensional lattice ($d_f = 1.89$)
(iii) Backbone of a two dimensional percolation cluster ($d_f = 1.6$)
(iv) Elastic backbone of a two dimensional percolation cluster ($d_f = 1.10$)
After the generation of the network, we have studied the various statistical features, viz., time or frequency distribution, degree distribution, correlation distribution and link-length distribution. We compare the results for the four different types of Euclidean spaces with the observations of real earthquake data and also examine whether the results at all depend upon the fractal dimensionality of the system.

We have simulated the earthquake network on all four kinds of Euclidean spaces with different system sizes pertaining to technical limitations. An averaging over a maximum of 1000 configurations have been used in all cases. The network generation procedure is as follows:
(i) Selection of nodes: For the continuous two dimensional Euclidean lattice, we take a unit square space where nodes occur randomly and each node is assigned coordinates x_1, x_2 where $0 \leq x_i \leq 1$.
In order to simulate networks on percolation clusters or its backbone or elastic backbone, we have taken a two dimensional square lattice where the occupation probability is equal to the percolation threshold, $p_c = 0.592746$ [12]. Once it is checked that a percolation cluster (with $d_f = 1.89$) does occur, nodes are chosen randomly on this cluster. The same has been done for the backbone with $d_f = 1.6$ and elastic backbone with $d_f = 1.10$ after identifying these from the percolation cluster.
(ii) Assignment of strengths: The selected sites represent the earthquake/aftershock epicenters or events and the epicenters are assigned strengths of magnitude m, randomly on a scale of 1 to 10.
(iii) Calculation of Distance (l_{ij}): When the epicenters with their strengths are chosen, we calculate the Euclidean distance l_{ij} between any two events/epicenters i and j where i and j also denote their time of occurrence.
(iv) Establishment of links: We start with the second event, $j = 2$ which is obviously linked with the only available preceding event, i.e., $i = 1$. For $j \geq 3$, events $i = 1, 2, 3, ..., j - 1$ are considered for the calculation of the correlation n_{ij} according to eq.(3) and node i is connected to node j for minimum n_{ij}.
The values of the constants used are $b = 1.0$ and $C = 10^{-9}$. While the maximum number of points used in the continuous $2d$ Euclidean space is $n = 5000$, for the percolation clusters we have used lattices of size $L \times L$ where L varies from 100 to 800. On the percolation clusters we have chosen from 500 to a maximum of 2000 sites as epicenters.

Statistical properties:
After the links have been established, we have evaluated the different distributions defined earlier. The results for these distributions do not show any significant finite size

effect in any of the fractals considered.

(I) Distribution of time interval between aftershocks: First we observe the nature of the distribution of time interval between events. This should follow the Omori law [6], according to which the distribution should have a power law decay with an exponent ~ 1.0. We in fact obtain a power law fall with the exponent close to 1.0 for all the four fractals (Fig. 1). For the elastic backbone a cutoff region is observed beyond $t \sim 100$.
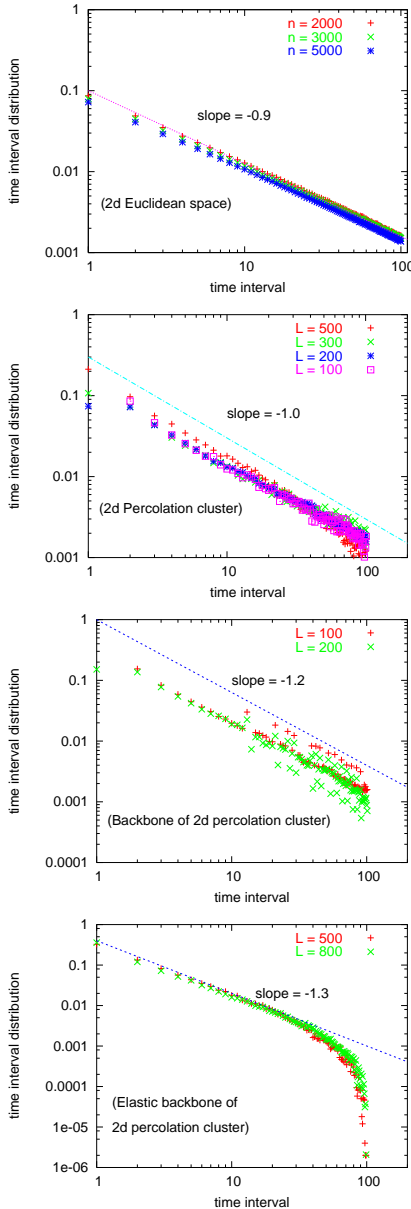
(II) The out-degree distribution: The earthquake network is a directed network [1,10] where the links are directed in time. The in-degree, i.e, the number of incoming links for any node is one, while the out-degree k of a node gives the number of aftershocks generated by that node. We observe that the number of nodes with out-degree k follows a power law decrease, $P(k) \sim k^{-\gamma}$ with $\gamma \sim 2$, for all four types of Euclidean spaces in agreement with the observations of [1]. Although the curves (Fig. 2) show occassional kinks, but an approximate straight line can be fitted for all the four cases.

FIG. 1. The distribution of time interval between events. The distribution follows a power law decay in all cases with exponent close to 1.0 which is in agreement with the Omori law. A cutoff is observed beyond $t \geq 100$ for the elastic backbone.
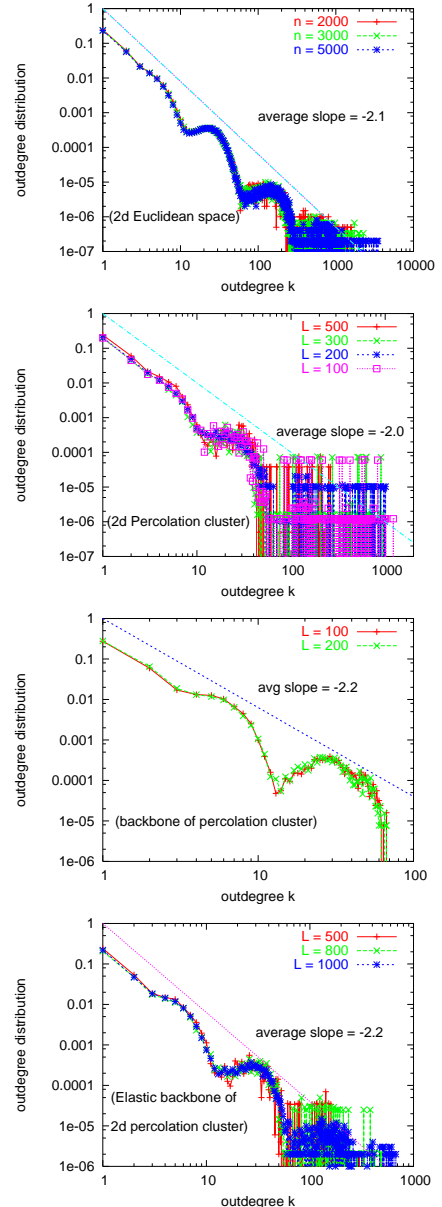
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FIG. 2. The out degree distribution for earthquake lattice simulated on the four kinds of spaces lattice as mentioned in the figures above. In each case it closely follows a power law decrease with average slope around 2.0.

(III) Correlation Distribution: The plot (Fig. 3) of the correlation distribution $\mathcal{N}(n_{ij})$ also has a power law decay, with a slope $\sim 2.6 \pm 0.2$ which deviates considerably from the observed value in [1] (this issue has been discussed later). A cutoff region exists here for all the cases.

tribution $\mathcal{L}(l_{ij})$ for the network grown on the different spaces does not follow a power law decay except for the elastic backbone of the two dimensional percolation cluster, with slope ~ 1.2. For the other three cases, i.e., the continuous Euclidean lattice, the two dimensional percolation cluster and the backbone of the percolation cluster, the distributions show a power law decay with an exponential cut off given by, $ax^{-\rho}\exp(-\lambda x)$ where a, ρ and λ are constants. The values of $\rho(\sim 0.1)$ and $\lambda(\sim 0.1)$ for the three cases are comparable; exact values are indicated in Fig. 4.
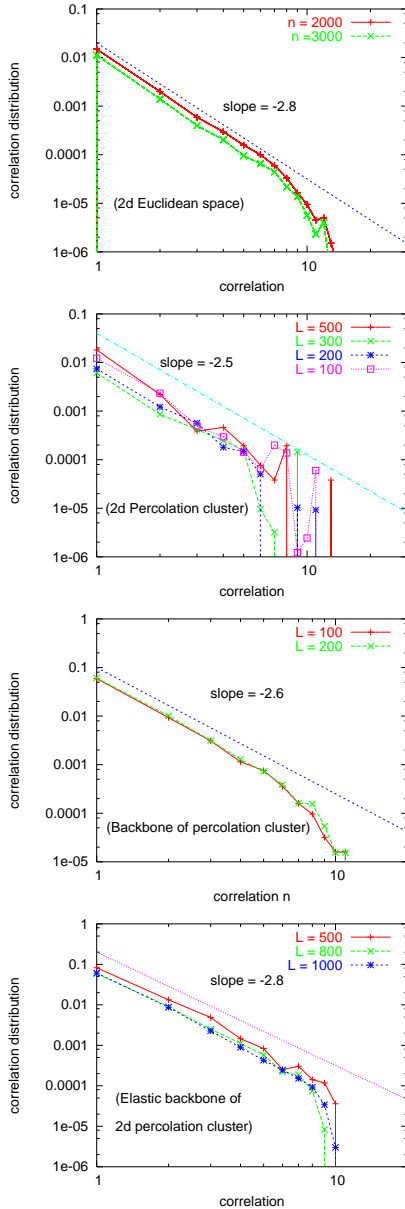

FIG. 3. The plots of the correlation distribution for the four types of fractals. Here the plot follows a power law decay in all cases with slope $\sim 2.6 \pm 0.2$.

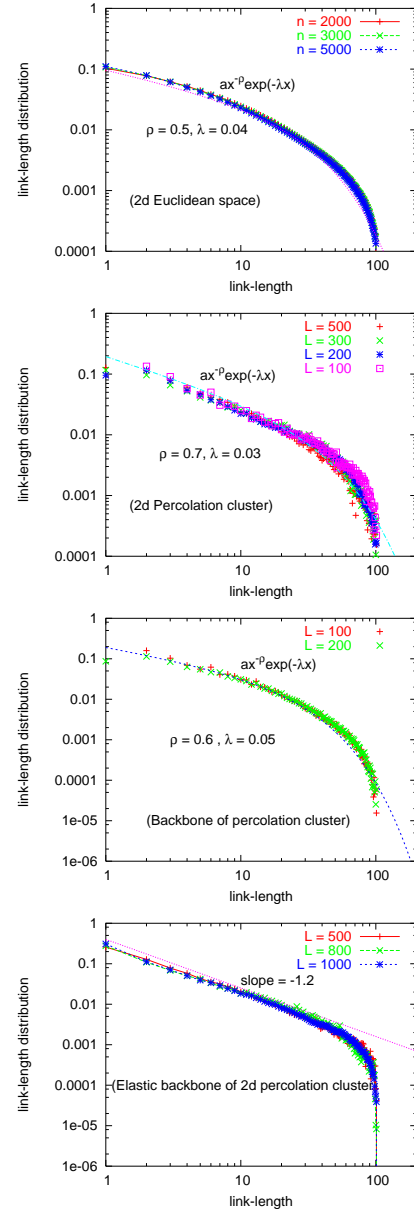(IV) Link length distribution: The link-length dis-

FIG. 4. The link length distributions for the four types of fractals.

To summarise the results, we have generated earthquake aftershock networks on Euclidean spaces with different fractal dimensions. The linking scheme follows the prescription of [1]. We have compared the distribution functions corresponding to the time intervals, out-degree, correlation and link-lengths from these networks. The results partially agree with the real data, e.g., the time interval distribution shows a power law decay as given by the Omori law, the out-degree distribution has an approximate power law decrease - in both cases the exponents also agree with the observed values quite well. The link-length distribution in [1] is approximately a power law while here it seems to have an exponential cutoff. For the correlation distributions the exponent value deviates appreciably from the observed one. While $\nu(t)$, $P(k)$ and $\mathcal{L}(l)$ can be estimated independently without a network formalism, n_{ij} is a measure directly linked to the networking scheme. That our result for $\mathcal{N}(n_{ij})$ does not agree quantitatively with that of the SCEDC data [1] may be due to the fact that epicenters have been assumed to occur randomly while in reality there is expected to be a correlation. However, the exact values of $\mathcal{N}(n_{ij})$ (which depend on the parameters chosen) may not be of much significance since the task is to link the epicenters with the minimum n_{ij}. The fact that we get good agreement for $\nu(t)$ and $P(k)$ and fairly good agreement for $\mathcal{L}(l)$ reflects this. On the theoretical side our studies show an interesting result that the behaviour of these distributions are almost independent of the underlying spatial structure. We have varied the fractal dimensionality from 2 to 1.1 and we do not notice any appreciable change in the behaviour of the different quantities, either qualitatively or quantitatively.

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